Langevin Algorithms for Markovian Neural Networks and Deep Stochastic Control

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We consider the following **Stochastic Optimal Control** (SOC) problem associated with a **Stochastic Differential Equation** (SDE):

$$\min_u J(u) := \mathbb{E}\left[\int_0^T G(X_t)dt + F(X_T)\right], \tag{1}$$

$$dX_t = b(X_t, u_t)dt + \sigma(X_t, u_t)dW_t, \;\; t \in [0, T] \tag{2}$$

- X_t: trajectory vector
- u_t: control vector
- $b(X_t, u_t)$: controlled drift vector
- $\sigma(X_t, u_t)$: controlled diffusion matrix
- W_t: Brownian motion (white noise process)

$\implies$ Optimize a functional of a trajectory of a SDE X_t through the control u_t, including a random noise that affects the evolution of the system.

Example: Resource Management

An oil drilling company has to balance the costs of extraction and of storage of oil in a volatile energy market:

- **Trajectory:** Volatile global oil price and quantity of stored (unsold) oil for the company
- **Control:** Quantities of instantaneously extracted, stored and sold oil

Figure: Offshore oil rig - Source: Unsplash

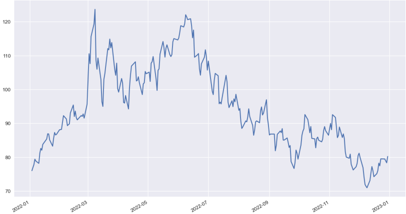


Figure: Crude oil price during the year 2022

Euler-Maruyama scheme

$$\min_{\theta} \bar{J}(\bar{u}_\theta) := \mathbb{E}\Big[\sum_{k=0}^{N-1}(t_{k+1}-t_k)G(\bar{X}^\theta_{t_{k+1}}) + F(\bar{X}^\theta_{t_N})\Big], \tag{3}$$

$$\begin{aligned}\bar{X}^\theta_{t_{k+1}} &= \bar{X}^\theta_{t_k} + (t_{k+1}-t_k)b(\bar{X}^\theta_{t_k}, \bar{u}_{k,\theta}(\bar{X}^\theta_{t_k})) \\ &\quad + \sqrt{t_{k+1}-t_k}\,\sigma(\bar{X}^\theta_{t_k}, \bar{u}_{k,\theta}(\bar{X}^\theta_{t_k}))\xi_{k+1},\end{aligned} \tag{4}$$

$\xi_k \sim \mathcal{N}(0, I_{d_2})$ i.i.d.

- **Time discretization** of $[0, T]$:

$$t_k := kT/N, \ k \in \{0, \dots, N\}, \quad h := T/N$$

- **Control** u with **parameter** θ using either one time-dependant neural network either N distinct neural networks: $u_{t_k} = \bar{u}_\theta(t_k, X_{t_k})$ or $u_{t_k} = \bar{u}_{\theta^k}(X_{t_k})$
- Since the process is **Markovian**, we assume the control depends only on the running position X_t (instead of the whole previous trajectory $(X_s)_{s\in[0,t]}$).

The parameter θ is optimized by **gradient descent**:

- Simulate batches of trajectories $\bar{X}$ depending on the Brownian motion.
- Compute $\nabla_\theta \bar{J} = \nabla_\theta \bar{J}(\bar{u}_{\theta_n}, (\xi_k^{i,n+1})_{1 \le k \le N})$; the gradient is computed by automatic differentiation as the gradient w.r.t. to θ is tracked all along the trajectory of the numerical scheme Giles and Glasserman (2005); Giles (2007)

In the literature:
SOCs are solved using specific techniques: Forward-Backward SDEs, Hamilton-Jacobi-Bellman (HJB) optimality conditions, stochastic dynamic programming. The resolution of SOCs by neural networks scales to the high dimension, contrary to dynamic programming Gobet and Munos (2005); Han and Weinan (2016); Bachouch et al. (2022); Laurière et al. (2023).

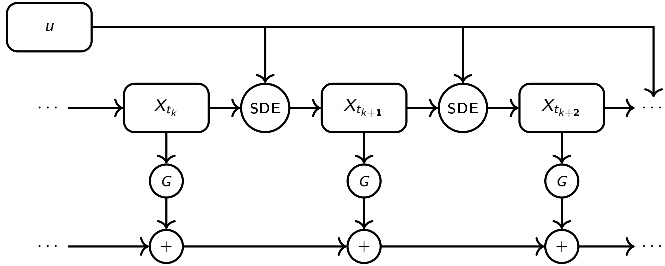


Figure: Markovian Neural Network with one control.

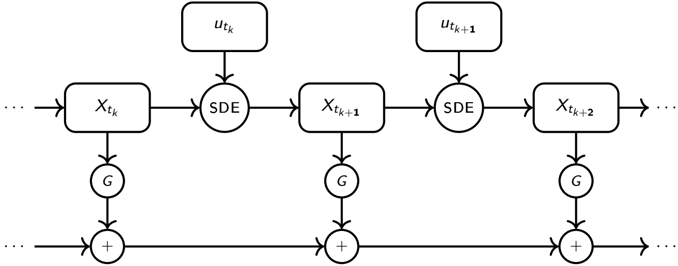


Figure: Markovian neural network with one control for every time step.

- If the control is applied at many discretization times, then the **Markovian Neural Network** becomes a **very deep** neural network, difficult to train directly.
- Adding noise during training is known to improve the learning procedure Neelakantan et al. (2015); Anirudh Bhardwaj (2019):

Gradient Langevin Algorithm

For some choice of **Preconditioner** rule P (Adam, RMSprop...), step size γ_{n+1} and and computed gradient g_{n+1}:

$$\theta_{n+1} = \theta_n - \gamma_{n+1} P_{n+1} \cdot g_{n+1} + \sigma_{n+1}\sqrt{\gamma_{n+1}}\mathcal{N}(0, P_{n+1}) \tag{5}$$

$\Longrightarrow$ per-dimension adaptive noise rate.

- Bras (2022): the deeper the network is, the greater are the gains provided by Langevin algorithms; introduces the **Layer Langevin** algorithm, consisting in adding Langevin noise only to the deepest layers.

$\Longrightarrow$ Analysis was conducted especially for deep architectures in **image classification**.

Our objectives

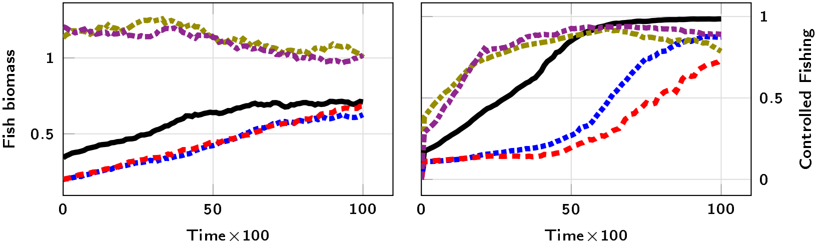
- Side-by-side comparison of non-Langevin/Langevin optimizers on different SOC problems: fishing quotas, financial hedging, energy management.
- If using multiple controls (second case), explore the benefits of Layer-Langevin.

Fish biomass $X_t \in \mathbb{R}^{d_1}$ with:

- Inter-species interaction κX_t
- Fishing following imposed quotas u_t
- Objective: keep X_t close to an ideal state $\mathcal{X}_t$.

Figure: Source: Unsplash

$$dX_t = X_t * ((r - u_t - \kappa X_t)dt + \eta dW_t)$$
$$J(u) = \mathbb{E}\left[\int_0^T (|X_t - \mathcal{X}_t|^2 - \langle \alpha, u_t \rangle)dt + \beta[u]^{0,T}\right]$$



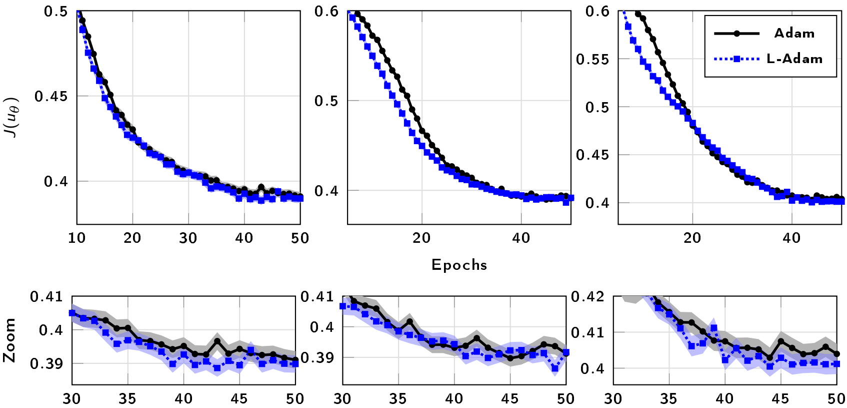


Figure: Comparison of Adam et L-Adam algorithms during the training for the fishing control problem with $N = 20, 50, 100$ respectively. J is estimated over 50×512 trajectories. A zoom on the last epochs is given.

Table: Best performance

	$N = 20$	$N = 50$	$N = 100$
Adam	0.3910	0.3912	0.4029
L-Adam	0.3886	0.3864	0.4011

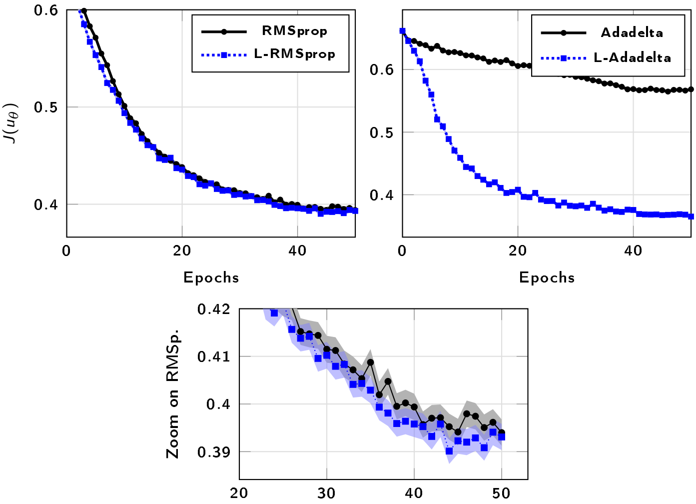


Figure: Comparison of Langevin algorithms with their non-Langevin counterparts during the training for the fishing control problem with $N = 50$.

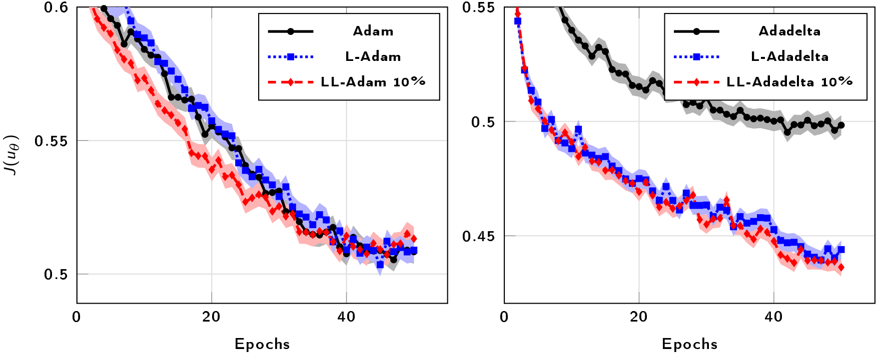


Figure: Training of the fishing problem with multiple controls with $N = 10$

We aim to replicate some payoff Z defined on some portfolio S_t by trading some of the assets with transaction costs; the control u_t is the amount of held assets. The objective is

Figure: Source: Unsplash

$$J(u) = \nu\left(-Z + \sum_{k=0}^{N-1} \langle u_{t_k}, S_{t_{k+1}} - S_{t_k} \rangle - \sum_{k=0}^{N} \langle c_{tr}, S_{t_k} * |u_{t_k} - u_{t_{k-1}}| \rangle\right) \quad (6)$$

where ν is a convex risk measure. We consider the assets S_t to be follow a Heston model and are tradable along with variance swap options.

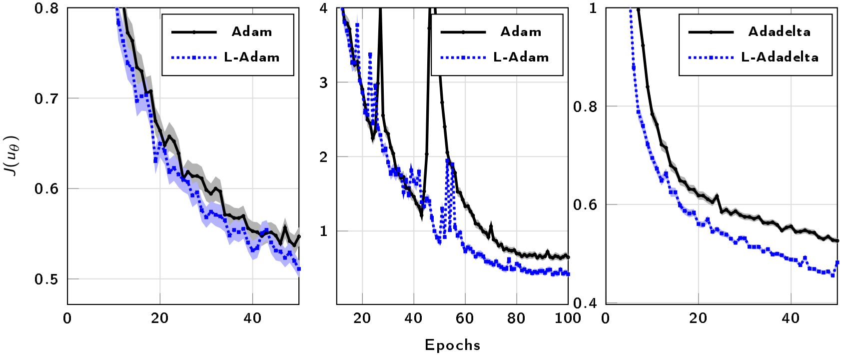


Figure: Comparison of algorithms during the training for the deep hedging control problem with $N = 30, 50, 50$ respectively

Table: Best performance

	Adam, $N = 30$	Adam, $N = 50$	Adadelta, $N = 50$
Vanilla	0.4448	0.6355	0.4671
Langevin	0.4306	0.4182	0.3773

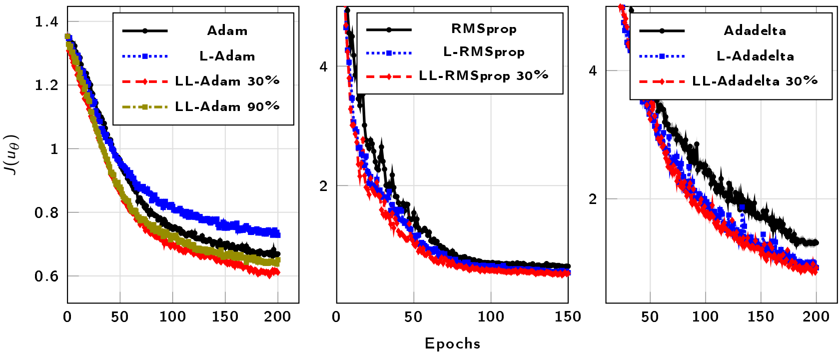


Figure: Training of the deep hedging problem with multiple controls with $N = 10$

Table: Best performance

	Adam	RMSprop	Adadelta
Vanilla	0.6626	0.5618	1.2900
Langevin	0.7278	0.4441	0.9250
Layer Langevin 30%	0.6004	0.4102	0.8554
Layer Langevin 90%	0.6377	–	–

An oil driller has to balance the costs of extraction E_t, storage S_t in a volatile energy market with oil price P_t:

$$dP_t = \mu P_t dt + \eta P_t dW_t$$

$$J(q) = -\mathbb{E}\left[\int_0^T e^{-\rho r} U\Big(q_r^v P_r + q_r^{v,s}(1-\varepsilon)P_r - (q_r^v + q_r^s)c_e(E_r) - c_s(S_r)\Big)dr\right],$$

$$E_t = \int_0^t (q_r^v + q_r^s)dr, \quad S_t = \int_0^t (q_r^s - q_r^{v,s})dr$$

where U is the utility function and $q_t = (q_t^v, q_t^s, q_t^{v,s})$ is the control (extracted, stored, sold from storage).

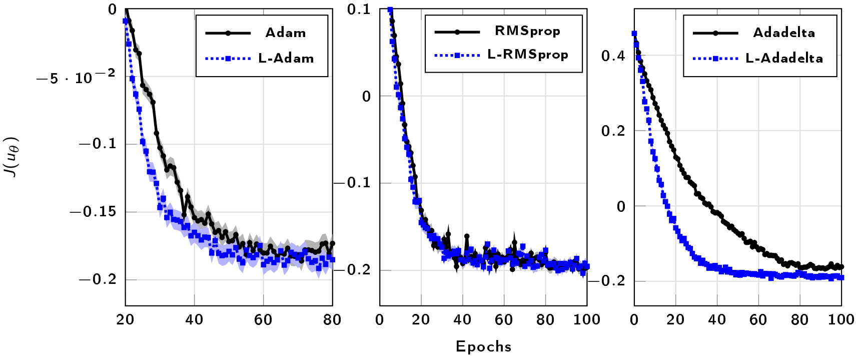


Figure: Comparison of algorithms during the training for the oil drilling control problem with $N = 50$

Table: Best performance

	Adam	RMSprop	Adadelta
Vanilla	-0.1729	-0.1985	-0.1649
Langevin	-0.1915	-0.2032	-0.1929

Conclusion

- In various problems, Langevin and Layer Langevin algorithms show improvements in comparison with their respective non-Langevin counterparts.
- Gains depend on the setting and optimizer; we observe that gains are limited or null for the RMSprop algorithm.
- For SOC with multiple controls, we proved the gains of Layer Langevin algorithms with a small number of layers ($\sim$10%-30%).

Thank you for your attention !

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